Random Graphs
Exercise Sheet 5

Question 1. Show that the hitting time for the existence of a matching in the bipartite random graph process is the same as the hitting time for having minimum degree at least one.

Question 2. Let r be fixed. Determine a threshold (ideally as in Theorem 6.9) for having a collection of r edge-disjoint perfect matchings in $G(n,p)$.

Question 3. Prove Claim 19 and Claim 20 from the proof of Theorem 6.13

Question 4. Show that if 3 divides n and $p = \omega\left(\sqrt{\frac{\log n}{n}}\right)$ then with high probability $G(n,p)$ can be covered by $\frac{n}{3}$ disjoint triangles.

Question 5. Show that Theorem 6.13 also holds for the bipartite random graph model $G(n,n,p)$

Question 6. Let $\left\{\{x_i, y_i\} : i \in \left[\binom{n}{2}\right]\right\}$ be an enumeration of the pairs $[x]^{(2)}$. Let D_i be a random digraph generated by the following procedure

- Independently for all $j \leq i$, both arcs $(x_j, y_j), (y_j, x_j) \in D_i$ with probability p and neither with probability $1-p$;
- Independently for $j > i$, each arc (x_j, y_j) and (y_j, x_j) is in D_i independently with probability p.

Demonstrate a coupling between $D_i \setminus \{(x_i, y_i), (y_i, x_i)\}$ and $D_{i-1} \setminus \{(x_i, y_i), (y_i, x_i)\}$. Show that for all i

$$\mathbb{P}(D_i \text{ has a directed Hamiltonian cycle}) \geq \mathbb{P}(D_{i-1} \text{ has a directed Hamiltonian cycle}).$$

Deduce that, for every n and p

$$\mathbb{P}(D(n,p) \text{ has a directed Hamiltonian cycle}) \geq \mathbb{P}(G(n,p) \text{ has a Hamiltonian cycle}).$$